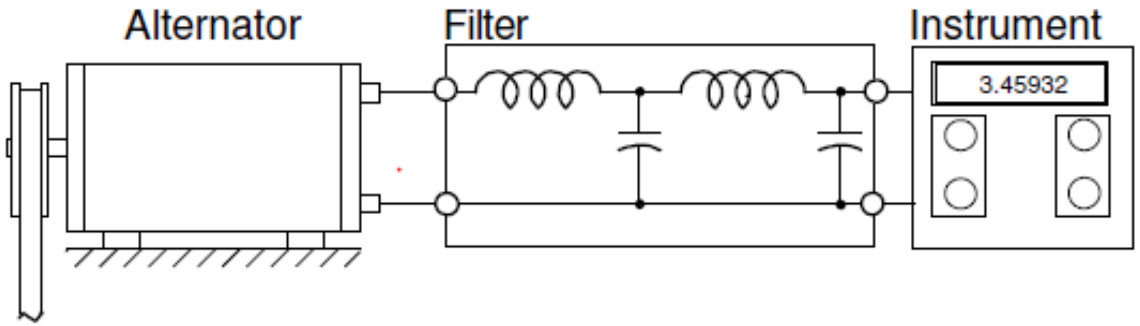
GRASP Laboratory
General Robotics & Autonomous Systems and Processes

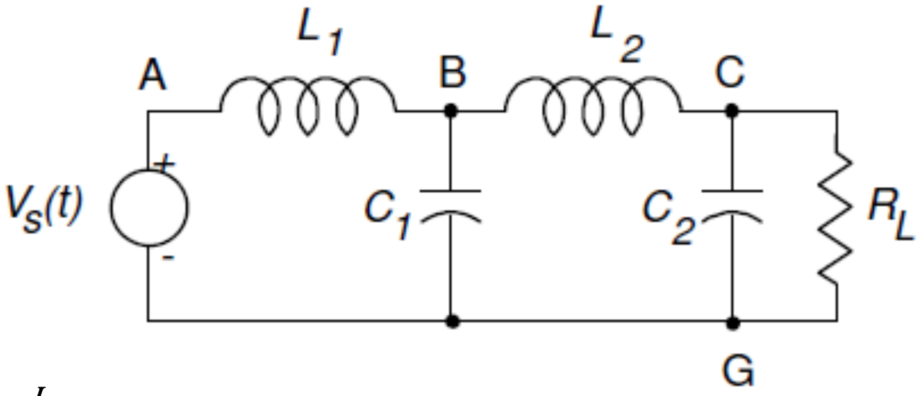
Mechatronic Modeling and Design with Applications in Robotics

Linear Graph Toolbox and Examples

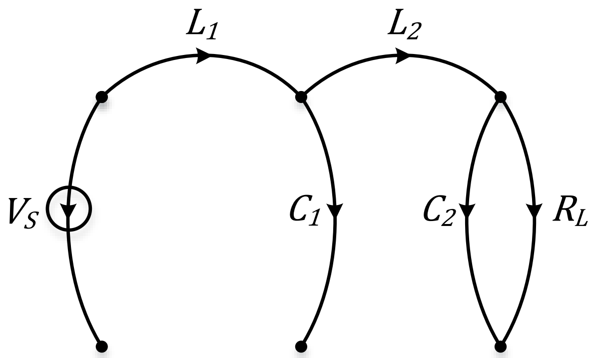
Linear Graph Vs. Bond Graph



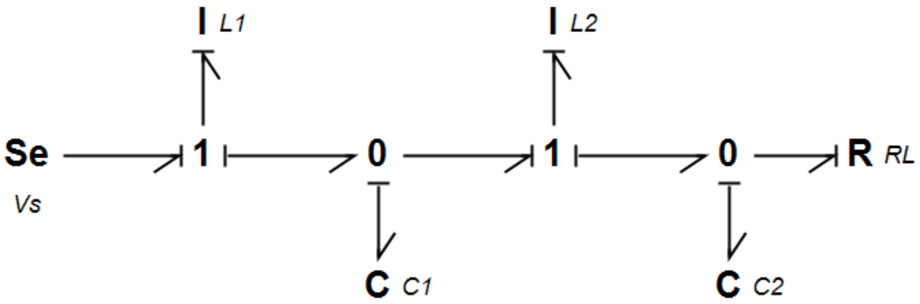
Physical System Model



Schematic Model of a Filter Circuit

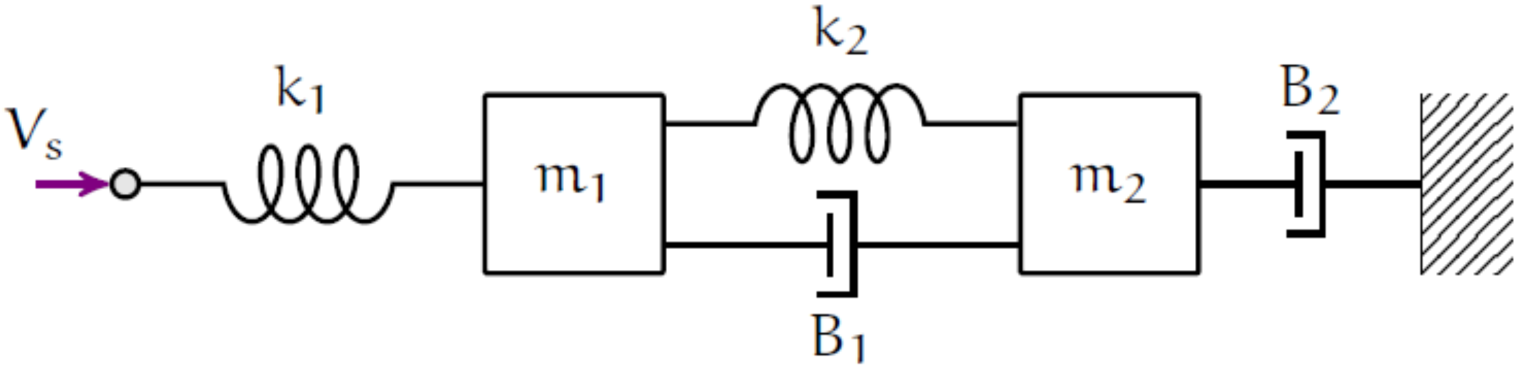


LG Model

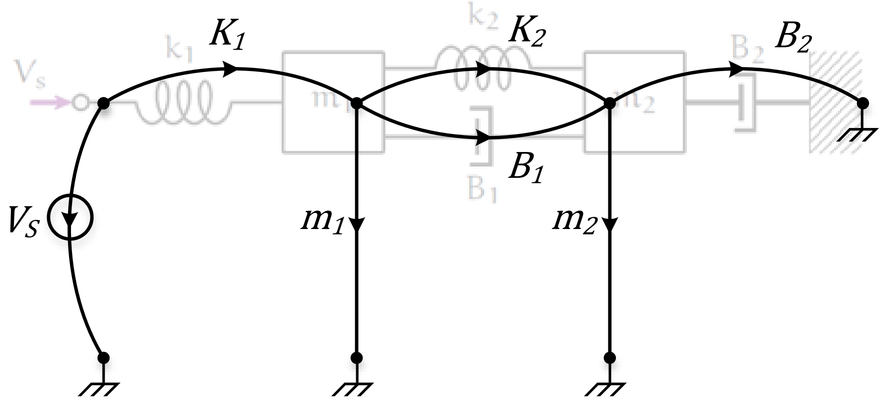


BG Model

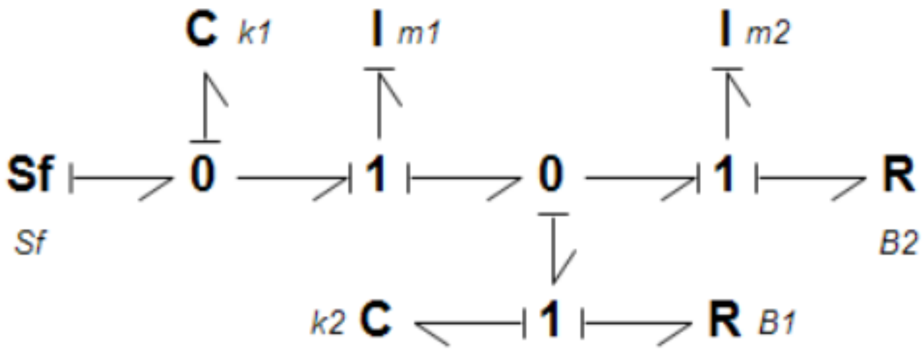
Another Example



Schematic Model of a Mass-Spring-Damper System



LG Model



BG Model

Summary

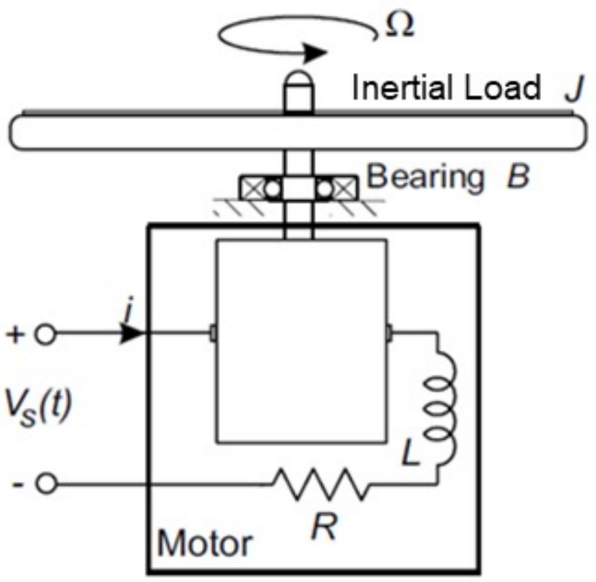
LG theory for the modeling of dynamic systems has some significant benefits over the BG modeling approach:

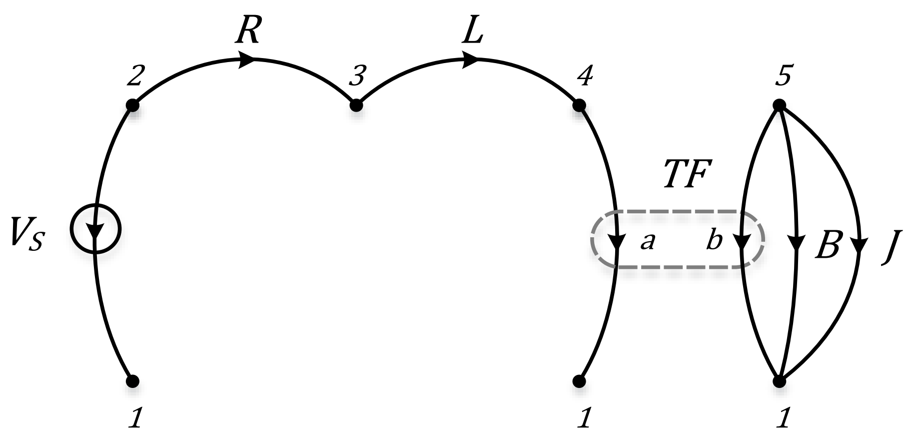
- The close resemblance which LG models have to their respective systems
- The intuitive nature in which these models can be constructed when compared to the BG approach
- The network-like representation of the LG method facilitates the analogous application of familiar node and loop equations commonly used in circuit analysis to systems outside of the electrical energy-domain
- The variables used as a result of the across and through analogy of the LG approach results in an easily understood state-space model consisting of common state-variable types, as opposed to the generalized displacements and momentums used in BG modeling

Index Values of Element Types and Energy Domains

Index	Element Type		Index	Energy Domain
1	Across-Variable Source		0	Generalized
2	A-Type Element		1	Electrical
3	Transformer		2	Mechanical Translational
4	Gyrator		3	Mechanical Rotational
5	D-Type Element		4	Hydraulic/Fluid
6	T-Type Element		5	Thermal
7	Through-Variable Source			

Example



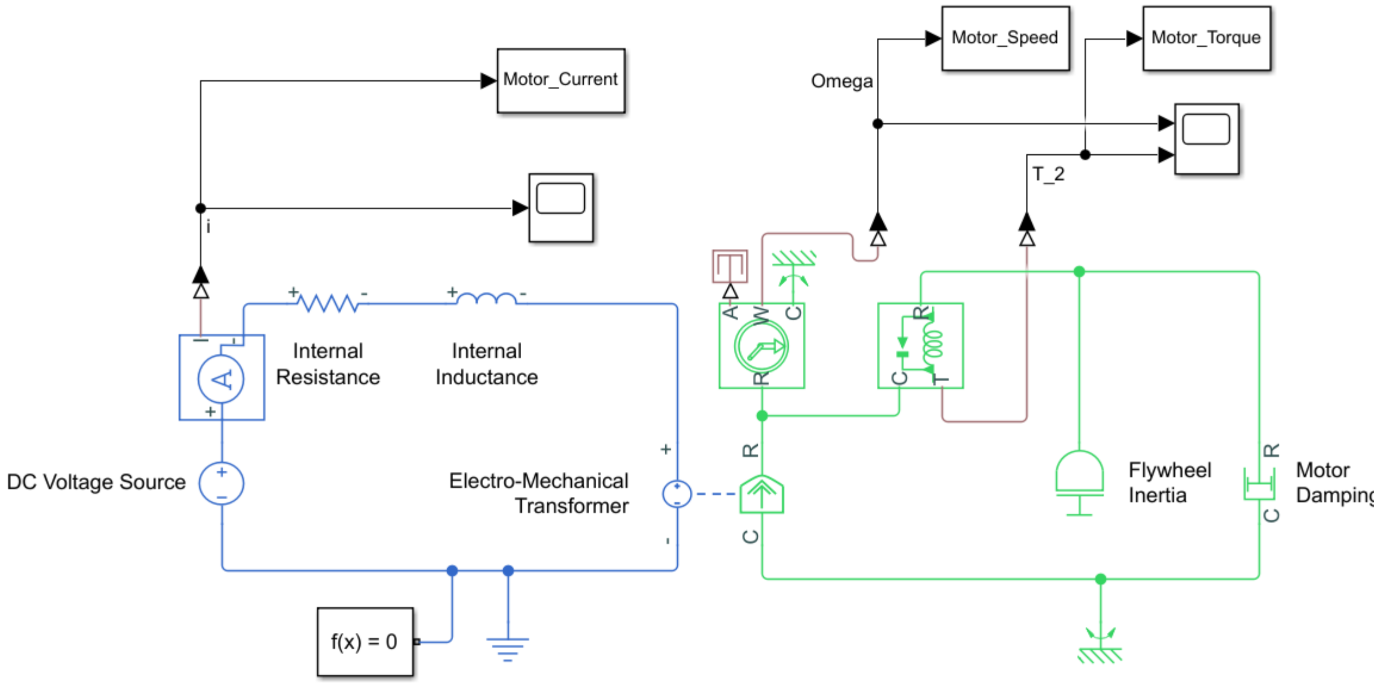


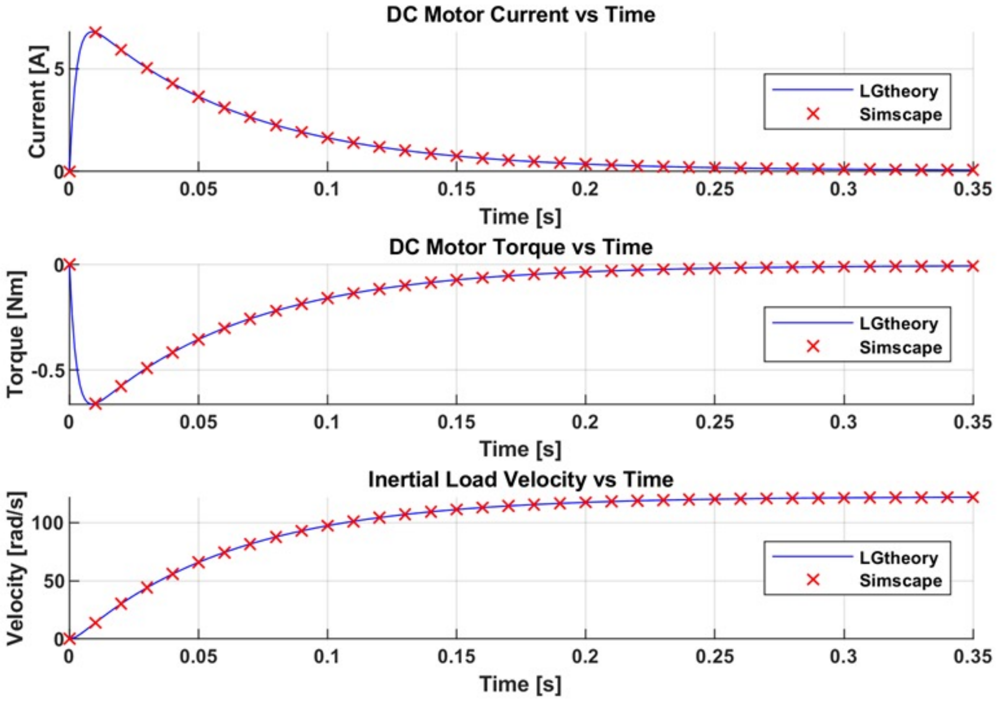

```
LG.S = [2 2 3 4 5 5 5];

LG.T = [1 3 4 1 1 1 1];

LG.Type = [1 5 6 3 3 5 2];

LG.Domain = [1 1 1 1 3 3 3];

syms s R L TFa TFb B J

LG.Var_Names = [s R L TFa TFb B J];

syms i_TFa(t) Tau_TFb(t) Omega_J(t)

LG.y = [i_TFa(t) Tau_TFb(t) Omega_J(t)];

[Model] = LGtheory(LG);
```

Functions

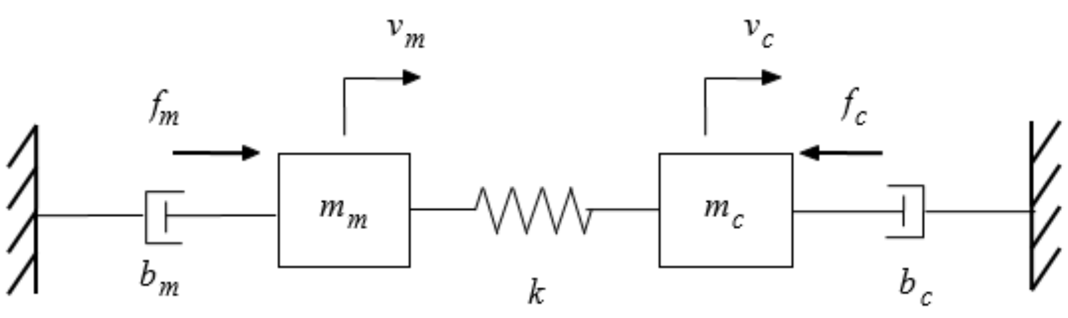
Check Model Inputs	CheckModel(LG);
Conversion to Incidence Matrix Representation	[Model] = IncidenceMatrix(LG);
Building the Normal Tree	[Model] = BuildNormalTree(LG,Model);
Variable Classification	[Model] = ClassifyVariables(LG,Model);
Constitutive Equations	[Model] = ElementalEquations(LG,Model);
Network Equations	[Model] = NetworkEquations(Model);
Creating the State-Space Matrices	[Model] = StateSpaceMatrices(LG,Model);
Standard State-Space Form Conversion	[Model] = StandardForm(Model);

Simscape Model of a DC Motor with an Inertial Load



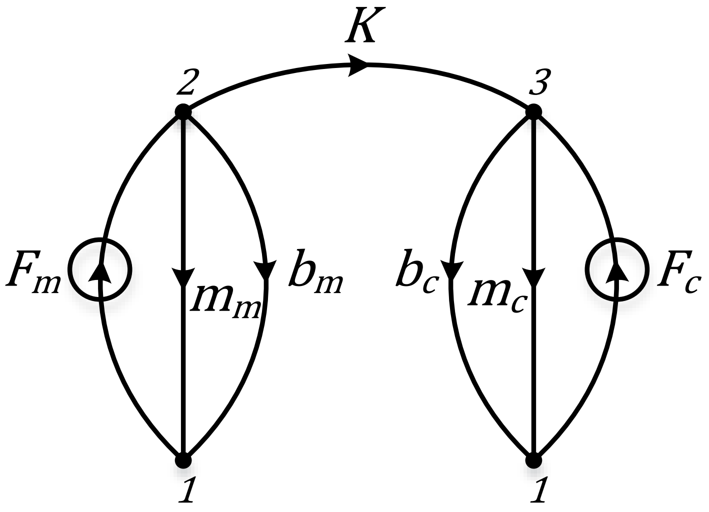


Example 1: Mechanical Translational System



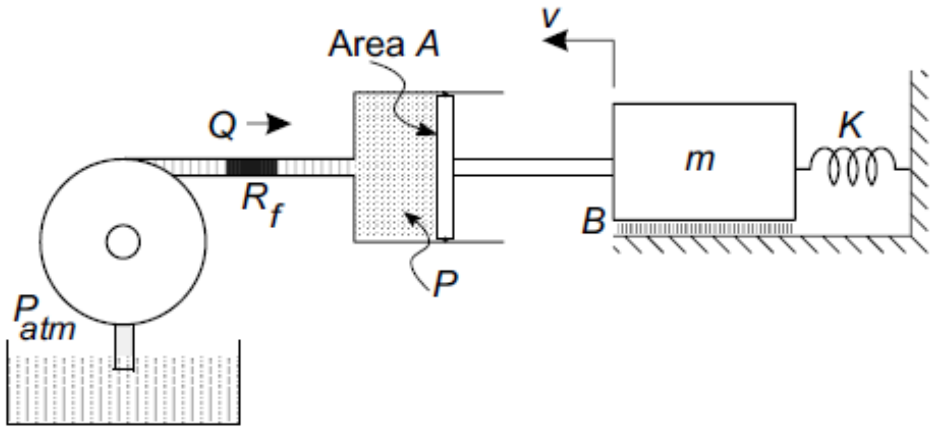
$$\begin{bmatrix} \dot{v}_{m_m} \\ \dot{v}_{m_c} \\ \dot{F}_K \end{bmatrix} = \begin{bmatrix} -\frac{b_m}{m_m} & 0 & -\frac{1}{m_m} \\ 0 & -\frac{b_c}{m_c} & \frac{1}{m_c} \\ K & -K & 0 \end{bmatrix} \begin{bmatrix} v_{m_m} \\ v_{m_c} \\ F_K \end{bmatrix} + \begin{bmatrix} \frac{1}{m_m} & 0 \\ 0 & \frac{1}{m_c} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_m \\ F_c \end{bmatrix}$$

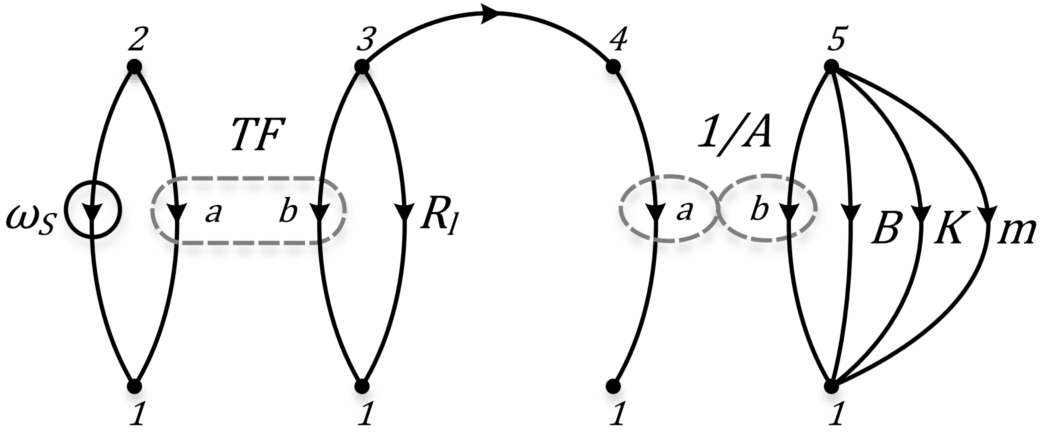
$$\begin{bmatrix} v_{m_m} \\ v_{m_c} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_{m_m} \\ v_{m_c} \\ F_K \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_m \\ F_c \end{bmatrix}$$




```
LG.S =        [1 2 2 2 3 3 1]; %Source vector
LG.T =        [2 1 1 3 1 1 3]; %Target vector
LG.Type =     [7 2 5 6 2 5 7]; %Type vector
LG.Domain =   [2 2 2 2 2 2 2]; %Domain vector
syms m m_m b_m K m_c b_c c
LG.Var_Names = [m m_m b_m K m_c b_c c];
syms v_m_m(t) v_m_c(t)
LG.y = [v_m_m(t) v_m_c(t)];
[Model] = LGtheory(LG);
```

Example 2: Hydro-mechanical System

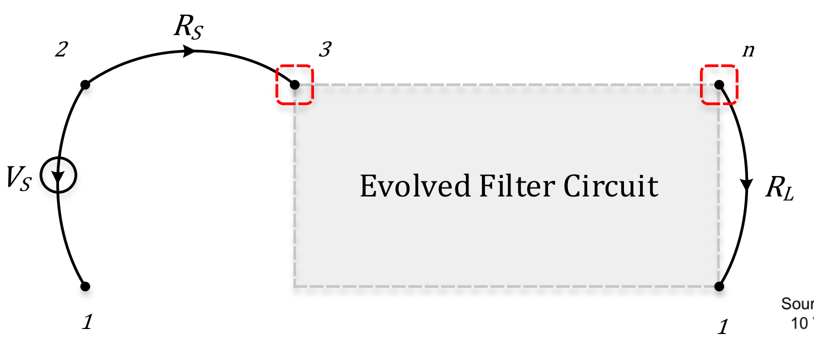


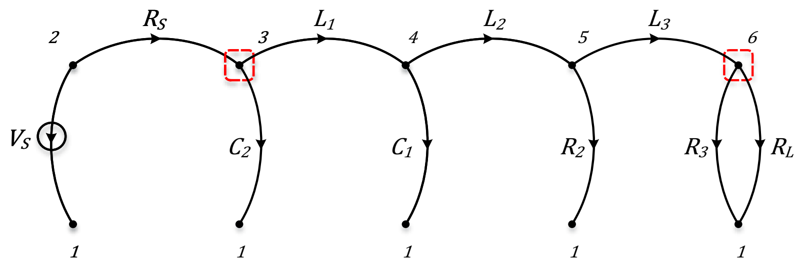


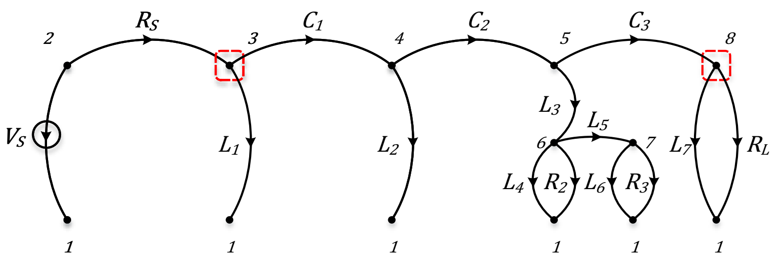

```
LG.S =       [2 2 3 3 3 4 5 5 5 5];

LG.T =       [1 1 1 1 4 1 1 1 1 1];

LG.Type =    [1 3 3 5 5 4 4 5 6 2];

LG.Domain =  [3 3 4 4 4 4 2 2 2 2];

syms s TF R_l R_f A B K m

LG.Var_Names = [s TF TF R_l R_f 1/A 1/A
B K m];

syms P_R_f(t) v_m(t)

LG.y = [P_R_f(t) v_m(t)];

[Model] = LGtheory(LG);
```

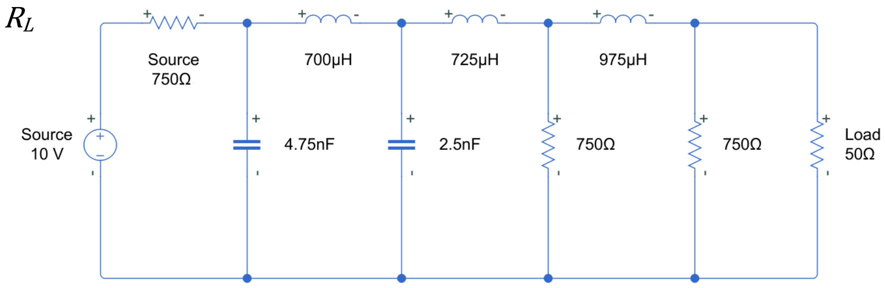
Circuit Design

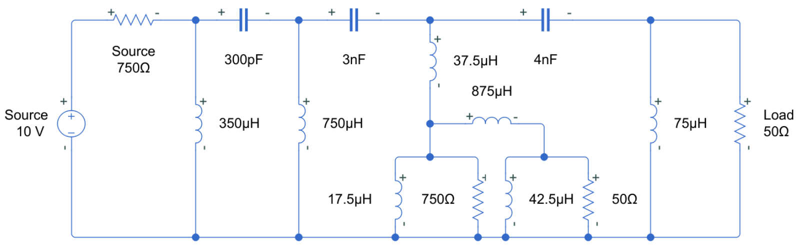
Embryo Model





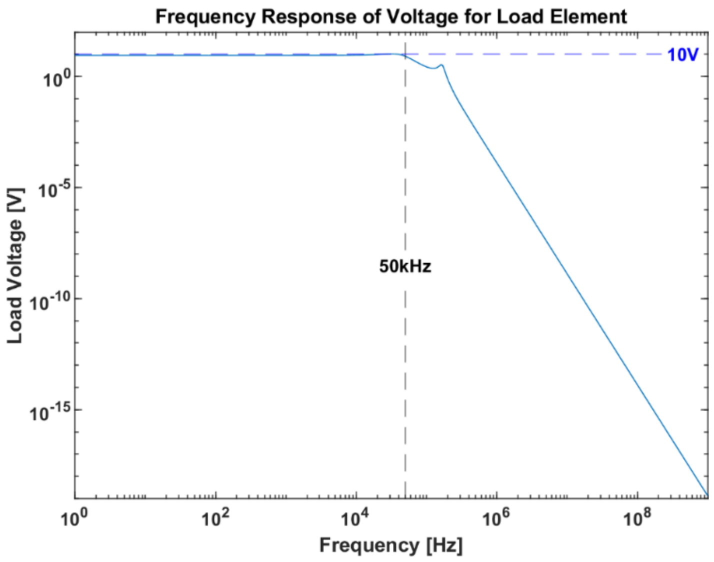


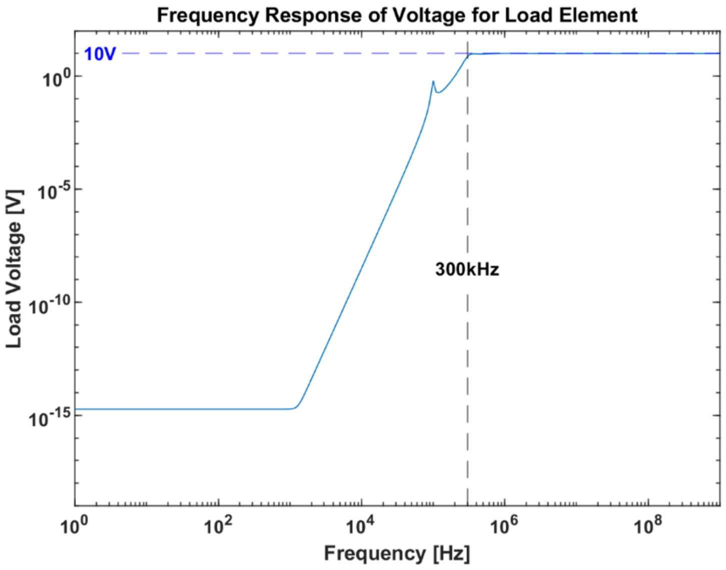




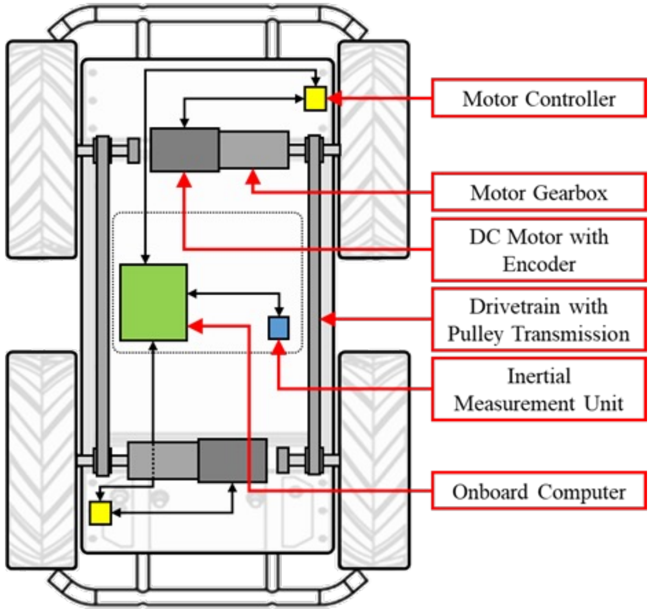
Matlab:

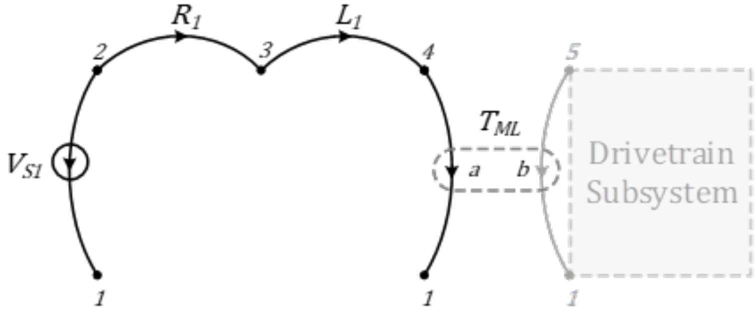
- LGtheory toolbox
- GP-based MATLAB toolbox developed by Sara Silva at the University of Coimbra



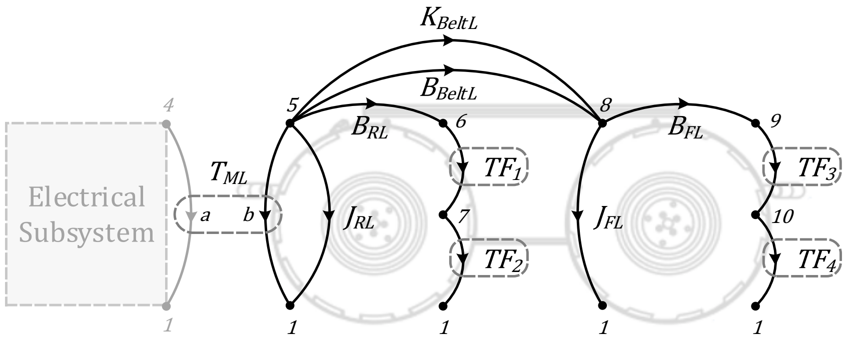


Mobile Robot

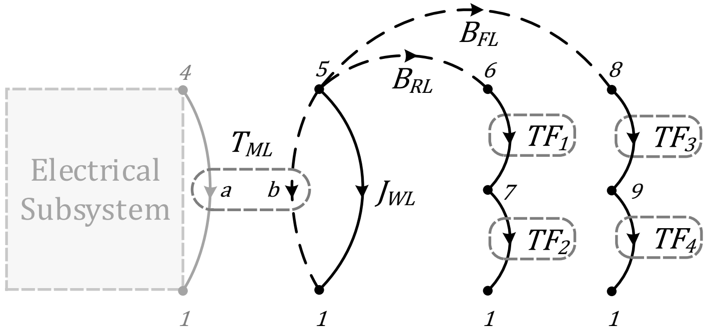




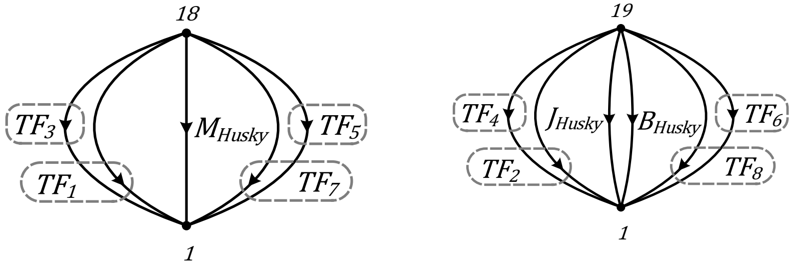
LG Model and the Normal tree of the Husky robot electrical subsystem.



left-side drivetrain subsystem overlaid on the profile

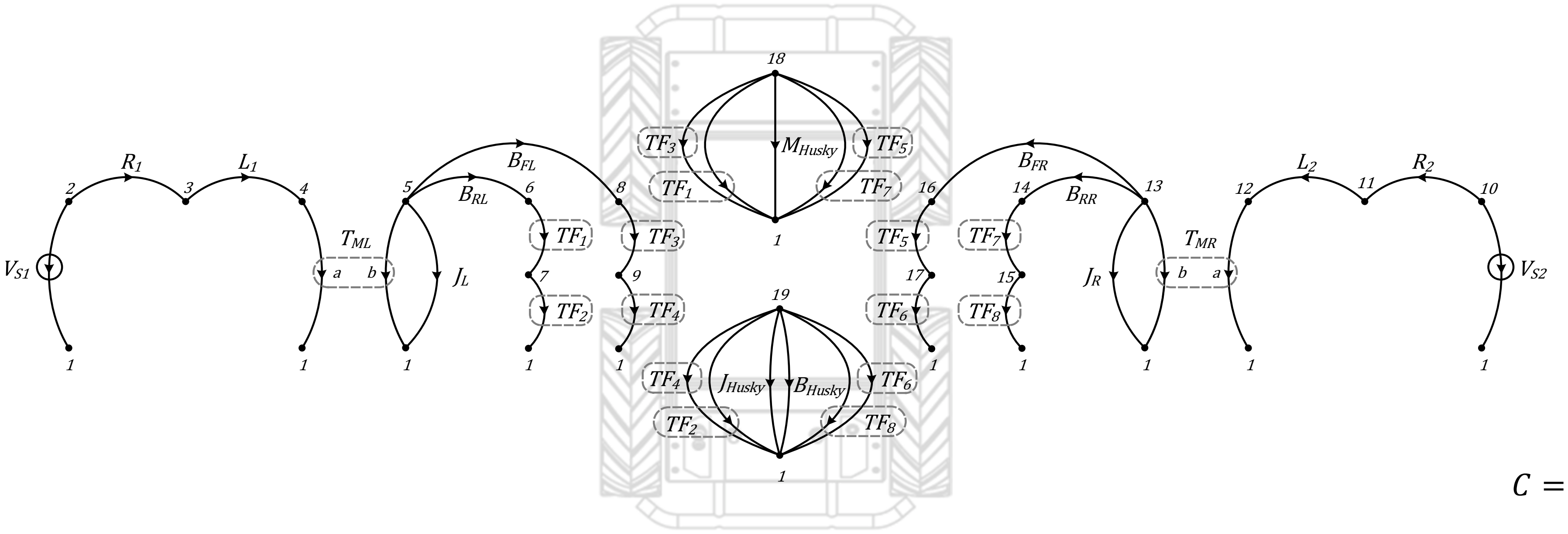


Simplified LG model and the normal tree of the left-side drivetrain subsystem.



Translational and rotational dynamic subsystem

Completed Model



$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \end{bmatrix}$$

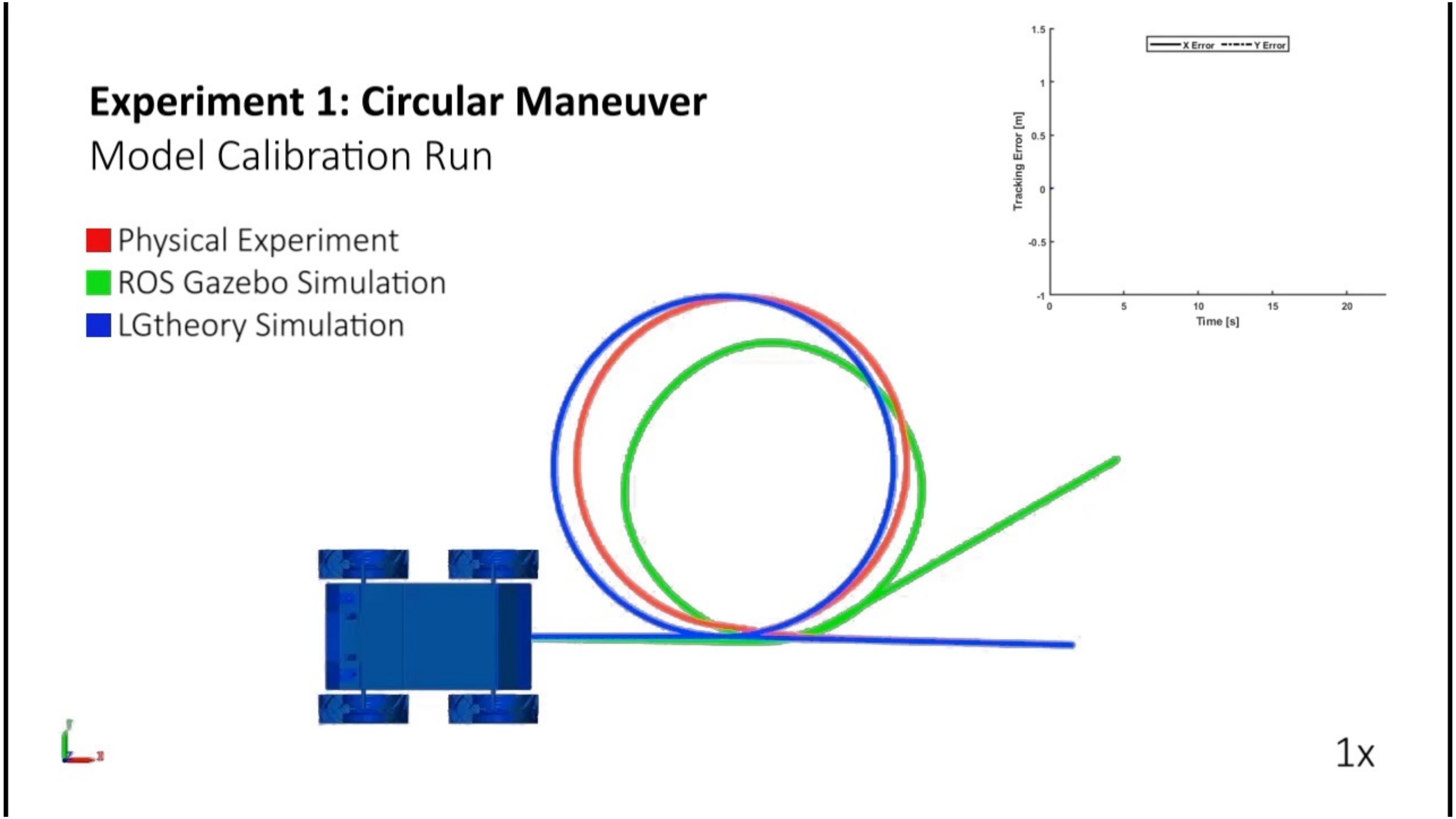
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = [0]_{4\times 2}$$

$$A = \begin{bmatrix} \frac{-B_{FL} - B_{RL}}{J_L} & 0 & \frac{B_{FL}TF_3 + B_{RL}TF_1}{J_L} & \frac{B_{FL}TF_4 + B_{RL}TF_2}{J_L} & \frac{T_{ML}}{J_L} & 0 \\ 0 & \frac{-B_{FR} - B_{RR}}{J_R} & \frac{B_{FR}TF_5 + B_{RR}TF_7}{J_R} & \frac{B_{FR}TF_6 + B_{RR}TF_8}{J_R} & 0 & \frac{T_{MR}}{J_R} \\ \frac{B_{FL}TF_3 + B_{RL}TF_1}{M_H} & \frac{B_{FR}TF_5 + B_{RR}TF_7}{M_H} & \frac{-B_{RL}TF_1^2 - B_{FL}TF_3^2 - B_{FR}TF_5^2 - B_{RR}TF_7^2}{M_H} & \frac{-B_{FL}TF_3TF_4 - B_{FR}TF_5TF_6 - B_{RL}TF_1TF_2 - B_{RR}TF_7TF_8}{M_H} & 0 & 0 \\ \frac{B_{FL}TF_4 + B_{RL}TF_2}{J_H} & \frac{B_{FR}TF_6 + B_{RR}TF_8}{J_H} & \frac{-B_{FL}TF_3TF_4 - B_{FR}TF_5TF_6 - B_{RL}TF_1TF_2 - B_{RR}TF_7TF_8}{J_H} & \frac{-B_{RL}TF_2^2 - B_{FL}TF_4^2 - B_{FR}TF_6^2 - B_{RR}TF_8^2 - B_H}{J_H} & 0 & 0 \\ -\frac{T_{ML}}{L_1} & 0 & 0 & 0 & -\frac{R_1}{L_1} & 0 \\ 0 & -\frac{T_{MR}}{L_2} & 0 & 0 & 0 & -\frac{R_2}{L_2} \end{bmatrix}$$

System Identification

Description	Parameter	Value	Units
Voltage Inputs	V_{s1}, V_{s2}	± 24	V
Internal Motor Resistance	R_1, R_2	0.46	Ω
Internal Motor Inductance	L_1, L_2	0.22	mH
Motor Torque Constant	k_t	0.044488	$N \cdot m/A$
Gear Ratio	GR	78.71 : 1	Gear Ratio
Motor Transformer Ratio	T_{ML}, T_{MR}	$k_t \times GR$	$N \cdot m/A$
Drivetrain Inertia	J_{LW}, J_{RW}	0.08	$kg \cdot m^2$
Drivetrain Damping	$B_{RL,FL,FR,RR}$	Unknown	$rad/(N \cdot m \cdot s)$
Power Conversion Transformer Ratios	TF_{odd}	$TF_{odd} = \frac{1}{r_W}$	
	TF_{even}	$TF_{even} = \pm cos(\theta_{W_i}) \cdot r_{C_i} \cdot \frac{1}{r_W}$	
Husky Mass	M_{Husky}	48.39	kg
Husky Rotational Damping	B_{Husky}	Unknown	$rad/(N \cdot m \cdot s)$
Husky Inertia	J_{Husky}	3.0556	$kg \cdot m^2$

Experimentation Result



The End!!